

XVII. *Some further Observations on Atmospheric Refraction.*

By Stephen Groombridge, Esq. F. R. S.

Read March 31, 1814.

IN my former paper on atmospheric refraction, communicated to the Royal Society by my late friend, Dr. MASKELYNE, I considered the few observations made below 80° of zenith distance, as not sufficiently to be depended on, for the computation of a general formula of refraction: and I therefore used η Ursæ Majoris ($78^{\circ} 10'$ zen. dis.) as the lowest star for that purpose. Having since applied the computed refraction from the formula thence obtained, to observations of stars below 80° , I have noticed, that such stars so corrected, appeared to be further from the zenith below the Pole, than they ought to have been, from the observations above the Pole: and therefore that the refraction was less at those distances from the zenith, than I had assumed. This has induced me, in the years 1811 and 1812, to make a course of observations of stars below the Pole, above 80° zenith distance; and as near to the horizon, as the trees in Greenwich Park would permit; these being higher than the level of my Observatory. It may also be remarked, that those stars in my former table below 80° , produce the co-latitude in excess; as a confirmation, that the same formula will not apply to those larger arcs, where, from the rapid increase of the tangents, a small error in the assumed quantity becomes more sensible. Although various hypotheses may be formed, from the known density and tem-

perature of the atmosphere ; and from these causes may be computed the effect they should have on a ray of light passing through the same : yet we must resort to observation, for the verification of the theory ; and reduce the quantity so found, to the most simple and convenient formula. I shall proceed to deduce, from this course of observations, such formulæ as will appear to result, for the computation of the refraction ; from the zenith, to the lowest star which I have observed : these may be considered as sufficient for the observation of the sun at the winter solstice, in high latitudes ; since those of the moon, from its great parallax, and the planets from their general invisibility, would probably not be attempted. Nevertheless, it is to be wished, as a matter of curiosity, or from which some useful deductions might be made, that in those Observatories, wherein from their elevated situations it might be practicable, the true quantity of refraction should be ascertained to the horizon.

Of all the formulæ for computing the mean refraction, that proposed and used by Dr. BRADLEY, is the most convenient and applicable for the practical astronomer. But as it is now acknowledged, that the numbers he had assumed for the coefficient of r (the refraction;) and of x (the quantity at 45°) were too small : their real values will appear to be the mean of several arcs, and such as I now propose to be adopted. I have found, that the same formula will serve to 87° of zenith distance ; possibly this might not happen in low situations, where the height of the vapours would form a greater angle with the horizon : yet in more elevated places, we may reasonably suppose, that a general formula might be carried nearly to the horizon.

In the two annexed tables are given the mean of the observed zenith distances of the several stars; and in the next column the mean refraction, computed from the formula which I proposed in my former paper; viz. tang. $\overline{z - 3.3625 r \times 58''}$, 1192: and which has been applied to these observations, corrected for the barometer and thermometer. In the following column, the error or difference is shewn, between the computed and real zenith distances; the assumed mean refraction, corrected by these errors, will give the mean refraction, which should have been applied. From these last quantities are deduced the respective values of (y) the coefficient of r , and of x : and from the mean of the sixteen stars, the resulting numbers will be $y = 3.6342956$, $x = 58.''132967$. Having therefore reduced the whole sixteen by these mean values of y and x ; the error or difference is noted, when compared with the corrected mean refraction which should arise from observation. On a review of these errors, and noticing the mean state of the thermometer for each star, the errors seem to indicate a small correction. I assumed in my former paper, that the refraction as affected by the thermometer, varied in an arithmetical ratio of ,0021 for each degree of FAHRENHEIT's scale; and the mean state to be at 45° for the thermometer without. Continuing the same mean state, and changing the ratio to ,0020, these errors will be affected $\frac{1}{10000}$ of the refraction, for each degree above and below 45° ; which being applied, will reduce the final error, as shewn in the last column.

In the same manner I proceeded to find the values of y and x , from the six lower stars, contained in the second table; but the respective values of both y and x were variable, and each

in a decreasing ratio. To discover the law of variation for each, would have been complex ; therefore retaining the value of x as general, I found y to vary as the minutes of each degree above $87^\circ \times .00462$; the co-efficient (y) of r when so reduced is given ; the mean refraction resulting, is contained in the following column ; and being corrected for the new factors of the thermometer, there remains the final error.

With a view to assist me in ascertaining whether the refractions were affected by local vapours, Dr. BRINKLEY has kindly communicated to me some observations of low stars ; which when reduced by the same formula do not materially differ from my own. 12 Can. Ven. at $87^\circ 2'$ and α Lyræ at $87^\circ 42'$ of which there are the greater number of observations, the former gives the same result within half a second, and the latter $1\frac{1}{2}''$.

Several of the fixed Observatories in Europe being situated in sufficiently high latitudes to obtain the elevation of the pole with much correctness ; we are thence enabled, by the circumpolar stars, to find the true quantity of refraction, for all zenith distances : and this having been so ascertained, we may apply the same to the observed zenith distance of the sun, at the winter solstice, as a test of its accuracy. With the smaller quantities of refraction, which were used by Dr. BRADLEY and others, fifty years since, it was not possible, that the latitude deduced from the elevation of the pole, and the mean of the solstices, could agree ; the distance of the pole from the equator, so computed, would be less than 90° . Hence also, the two solstices would shew an error of double that difference in the obliquity of the ecliptic, when obtained from the greatest and least zenith distances of the sun. The small number of

observations I have been able to make of the sun, to ascertain the agreement of the two solstices, are subjoined; these appear to confirm the refraction, as deduced from the circum-polar stars.

In the course of my observations I have noticed, that applying the correction for the thermometer without, gives the most accurate result. The difference is very sensible in great zenith distances, from the greater quantity of refraction; and we may reasonably infer, that when the front shutter of the Observatory is opened, the horizontal current of the air is of the same temperature as without; although from the short time of the shutter being opened, during the observation, it is not indicated by the thermometer in the telescope. I have therefore constantly used the thermometer without, for the correction, when I have opened the front shutter; but on all other occasions, I have applied the correction for the thermometer within. The instrument is protected from the horizontal current of air, when the sloping shutters in the roof only are opened; the front shutters being five feet above the graduated circle.

Having formerly proposed certain factors for the thermometrical correction of the refraction, and now finding them relatively the same for the thermometer within and without, changing the ratio for each degree of FAHRENHEIT $\frac{1}{10000}$; I shall adopt, hereafter, the following formula. Putting h° for the degree of the scale; then for the thermometer within, $49^{\circ} - h^{\circ} \times .0023$ when below the mean; $49^{\circ} - h^{\circ} \times .0022$ when above the mean: and for the thermometer without, $45^{\circ} - h^{\circ} \times .0020$; will produce the respective factors. When h° is less than the mean, these will be positive; when greater

than the mean, negative. These factors of different values will be nearly the same for the thermometer within and without, in summer; when the temperatures approximate: but in winter, when the temperatures may differ 6° or 8° , the factors will vary accordingly. It may therefore not be material, when the quantity of refraction is small, not exceeding $1\frac{1}{2}''$, whether the correction is applied for the thermometer within or without; but when the temperature within is supposed to be affected by the horizontal current from without, I should recommend, in all such cases, the use of the latter correction.

To make a direct comparison of these refractions with the French tables, it may be objected, that the latter being computed for the metrical barometer 0,760 m., which is equal to 29,93 inches of our barometer, the mean refraction from the proposed formula should be increased by the factor +,0111; since we reckon the mean state at 29,60 inches: but, the metrical thermometer at + 10 being equal to ours at 50° ; and the mean state being determined from observation, to be 45° of FAHRENHEIT; the factor for 50° will be — ,0100; which will nearly compensate the factor for the barometer. I have therefore compared the mean refractions resulting from my formulæ, with those of the French tables, from 80° to 90° : excepting the two last, the difference is not considerable; and whether these arise from defect in the formulæ, or from local causes, can only be determined by observation.

On comparing the refractions, now proposed, with my former deductions, they will not differ one second at 81° . At 72° the present is $\frac{1}{10}$ of a second less than the former; and at $74^{\circ} 50'$ the zenith distance of the winter solstice, $0.^{\prime}18$ must be deducted as a correction. It will appear on inspection of

the second table, that β Persei at $88^\circ 1'$ requires but a small equation ($-2'', 41$) when computed from my former numbers; and at 88° both these formulæ will coincide. I had therefore, in my former paper, determined too hastily, that the formula proposed would agree with observation so far as 88° ; not having then discovered the discrepancies between 80° and 88° , which are corrected by the present formula.

In order to facilitate the computation of the true refraction, it is required to form a table of mean refractions from certain formulæ. This may appear difficult at the first view; since every refraction must become equal to, $\text{tang. } \overline{z} - \overline{yr} \times x$; and unless r is assumed very nearly, several operations may be necessary before the differences will vanish. However, proceeding in small arcs of $10'$ to 70° , of $5'$ to 86° , of $4'$ to 88° , of $3'$ to 89° , and of $2'$ thence to the horizon, the second differences of the variation of these arcs may always be taken by inspection: and the resulting refraction will be equal to that assumed, in one operation. The correction for the barometer and thermometer will be the sum of the two factors in the annexed tables, into the mean refraction: and the product added thereto, according to the algebraic sign of their sum, will give the true refraction. This method is more expeditious, than by logarithms; no other tables of reference being required: and the computation will be effected with a small number of figures; which is an object I have constantly had in view.

S. GROOMBRIDGE.

Blackheath, 31 January, 1814.

Reduction of the Solstices.

Zen. dis. corrected.				Zen. dis. corrected.			
1810.				1812.			
Dec. 14	-	74° 55' 53.96"		June 7	-	28° 6' 25.76"	
26	-	-	52.53	14	-	-	23.05
27	-	-	49.75	15	-	-	23.74
30	-	-	52.66	29	-	-	23.04
			<hr/>	July 5	-	-	23.42
Nutation	+ 9.60	74° 55' 52.23"		Nutation	- 8.56	28° 0' 23.80	
Parallax	- 8.59			Parallax	- 4.04	-	13.61
Sun's lat.	+ 0.28	+ 1.11		Sun's lat.	- 1.01		
Corr. refr.	- 0.18						
Latitude	74° 55' 53.34			Latitude	28° 0' 10.19		
	51° 28' 2.18				51° 28' 2.18		
Mean obliqu. eclips.	23° 27' 51.16			Mean obliqu. eclips.	23° 27' 51.99		
1811.				1812.			
Dec. 9	-	74° 55' 53.64		Dec. 7	-	74° 55' 57.51	
16	-	-	52.11	8	-	-	50.51
22	-	-	49.79	11	-	-	55.93
23	-	-	55.20	13	-	-	54.00
25	-	-	50.96				
1812 30	-	-	52.30	Nutation	+ 7.68	74° 55' 54.64	
Jan. 2	-	-	55.41	Parallax	- 8.59		
			<hr/>	Sun's lat.	- 0.52	-	1.61
Nutation	+ 9.16	74° 55' 52.77"		Corr. refr.	- 0.18		
Parallax	- 8.59						
Sun's lat.	- 0.17	+ 0.22					
Corr. refr.	- 0.18						
Latitude	74° 55' 52.99			Latitude	74° 55' 53.03		
	51° 28' 2.18				51° 28' 2.18		
Mean obliqu. eclips.	23° 27' 50.81			Mean obliqu. eclips.	23° 27' 50.85		

TABLE I.

Star.	No. Obs.	Observed Zenith Distance.	Mean Refraction.	Error from Observ.	Mean Refrac, correc.	Error from the coeff. of r_1 .	Error from the Refrac at 45°.	Tang $\frac{x-y}{x+y}$	Error.	Therm. correct.	Final Error.
γ Herculis	15	39 30.08	6 19.96	+ 1.06	6 18.90	+ 0.0	- 3.62402	58.13601	' 18.84	" 0.06	" 0.28
Capella	30	82 37 49.50	7 6.07	+ 1.12	7 4.95	+ 0.2	- 3.55849	58.13114	7 4.49	- 0.46	+ 0.26
ψ Ursæ maj.	14	82 53 1.76	7 19.80	+ 1.65	7 18.15	+ 0.0	- 3.62441	58.13499	7 18.11	- 0.04	- 0.26
β Aurigæ	33	83 29 36.28	7 56.66	+ 2.01	7 54.65	+ 0.1	- 3.61064	58.13373	7 54.45	- 0.20	+ 0.56
α Cygni	29	83 47 18.26	8 16.55	+ 2.61	8 13.94	+ 0.0	- 3.65160	58.13613	8 14.09	+ 0.15	- 0.10
ω Ursæ maj.	10	84 12 4.14	8 47.17	+ 4.08	8 43.09	+ 0.2	- 3.74129	58.14097	8 44.21	+ 1.12	- 0.68
π Persei	11	84 15 26.81	8 51.62	+ 3.04	8 48.58	+ 0.5	- 3.63505	58.13429	8 48.58	0.00	+ 0.42
λ Ursæ maj.	10	84 31 48.93	9 14.14	+ 4.30	9 9.84	+ 0.6	- 3.70809	58.13833	9 10.72	+ 0.88	- 0.50
δ Aurigæ	13	84 50 58.25	9 42.81	+ 2.64	9 40.17	+ 0.5	- 3.54666	58.12771	9 38.85	- 1.32	+ 0.81
η Lacertæ	15	85 4 10.55	10 4.18	+ 4.37	9 59.81	+ 0.7	- 3.63439	58.13276	9 59.79	- 0.02	- 0.50
ϵ Andromedæ	11	85 4 32.79	10 4.79	+ 4.48	10 0.31	+ 0.2	- 3.64000	58.13328	10 0.40	+ 0.09	- 0.09
ζ Cygni	14	85 10 51.26	10 15.52	+ 4.83	10 10.69	+ 0.9	- 3.64778	58.13364	10 10.91	+ 0.22	- 0.37
μ Ursæ maj.	12	85 53 46.42	11 38.53	+ 7.02	11 31.51	+ 0.6	- 3.65335	58.13204	11 31.95	+ 0.44	- 0.62
δ Andromedæ	6	86 6 22.45	12 6.71	+ 7.41	11 59.30	+ 0.3	- 3.63706	58.13029	11 59.38	+ 0.08	- 0.14
γ Andromedæ	14	86 53 26.64	14 11.52	+ 10.42	14 1.10	+ 0.5	- 3.61333	58.12594	14 0.29	- 0.81	+ 0.03
α Andromedæ	6	86 58 29.91	14 27.05	+ 11 32.14	15.73	+ 0.2	- 3.62255	58.12617	14 15.27	- 0.46	- 0.13

TABLE II.

β Bootis	10	87 8 19.70	14 58.62	+ 12.30	14 46.32	+ 14	- 3.59582	14 47.49	+ 1.17	- 0.98	+ 0.19
γ Aurigæ	11	87 19 9.83	15 35.64	+ 7.24	15 28.40	+ 0.0	- 3.55576	15 25.91	- 2.49	+ 1.40	- 1.09
Aurigæ	15	87 29 7.86	16 11.89	+ 6.18	16 5.71	+ 5.7	- 3.49971	16 3.85	- 1.86	+ 1.16	- 0.70
Persei	16	88 1 0.03	18 23.73	- 2.41	18 26.14	+ 5.4	- 3.35247	18 24.77	- 1.37	+ 1.00	- 0.37
Cygni	5	88 31 1.12	20 53.78	- 16.0	21 9.78	+ 3.9	- 3.21379	21 11.37	+ 1.59	- 0.76	+ 0.83
Persei	5	88 42 35.53	21 59.32	- 28.78	22 28.1C	+ 5.3	- 3.16032	22 26.48	- 1.62	+ 1.08	- 0.54

Mean Refraction.

Zen. dis.	French tables.	S. G.	Value of y .
°	' "	' "	
80	5 19.8	5 19.18	3.63429
81	5 53.5	5 52.83	—
82	6 34.4	6 33.79	—
83	7 24.7	7 24.63	—
84	8 29.9	8 29.13	—
85	9 54.3	9 53.03	—
86	11 48.3	11 45.27	—
87	14 28.1	14 19.80	3.63429
88	18 22.2	18 19.83	3.35799
89	24 21.2	24 32.94	3.07989
90	33 46.3	34 28.13	2.80269

The two formulæ compared.

Zen. dis.	3.3625r x 58".1192	3.63429r x 58".132967
°	' "	' "
70	2 38.41	2 38.34
71	2 47.31	2 47.23
72	2 57.13	2 57.03
73	3 8.04	3 7.92
74	3 20.22	3 20.07
75	3 33.92	3 33.73
76	3 49.44	3 49.21
77	4 7.19	4 6.89
78	4 27.68	4 27.30
79	4 51.59	4 51.09
80	5 19.85	5 19.18
81	5 53.74	5 52.83
82	6 35.06	6 33.79
83	7 26.46	7 24.63
84	8 31.85	8 29.13
85	9 57.27	9 53.03
86	11 52.21	11 45.27
87	14 31.75	14 19.80
88	18 19.19	18 19.83

Mean Refraction calculated at $58^{\circ}, 132967$ for 45 Degrees of apparent Zenit

t Zenith Distance \times Tang. Z — 3.6342956r so far as 87° Zen. Dis. below which r is reduced

Zen. Dis.	Refrac.	Diff.	Zen. Dis.	Refrac.	Diff.	Zen. Dis.	Refrac.	Diff.	Zen. Dis.	Refrac.	Diff.	Zen. Dis.	Refrac.	Diff.	Zen. Dis.	Refrac.	D				
0° 0' 0" 58.01	" 0.34	54° 0' 1" 19.78	" 0.49	63° 0' 1" 53.53	" 0.82	71° 0' 2" 47.23	" 0.78	75° 30' 3" 41.22	" 1.30	80° 0' 5" 19.18	" 2.	10° 0' 0" 58.35	" 0.34	10° 1" 20.27	" 0.49	10° 1" 54.35	" 0.82	10° 5" 21.75	" 2.		
20° 0' 0" 58.69	" 0.34	20° 1" 20.76	" 0.49	20° 1" 55.17	" 0.82	10° 2" 48.80	" 0.79	40° 3" 42.52	" 1.31	10° 5" 24.36	" 2.	30° 0' 0" 59.03	" 0.34	30° 1" 21.25	" 0.49	30° 1" 56.00	" 0.83	15° 5" 27.01	" 2.		
40° 0' 0" 59.38	" 0.35	40° 1" 21.75	" 0.50	40° 1" 56.85	" 0.85	20° 2" 50.39	" 0.80	45° 3" 45.15	" 1.32	20° 5" 29.71	" 2.	50° 0' 0" 59.72	" 0.34	50° 1" 22.26	" 0.51	50° 1" 57.70	" 0.86	25° 5" 32.44	" 2.		
60° 0' 0" 59.72	" 0.35	50° 1" 22.26	" 0.51	50° 1" 57.70	" 0.86	25° 2" 51.20	" 0.81	55° 3" 47.84	" 1.37	80° 0' 5" 19.18	" 2.	60° 0' 0" 0.07	" 0.35	55° 1" 22.77	" 0.51	64° 0" 1" 58.56	" 0.87	76° 0" 3" 49.21	" 1.38	30° 5" 35.21	" 2.
10° 1" 0.42	" 0.35	10° 1" 23.28	" 0.51	10° 1" 59.43	" 0.89	35° 2" 52.83	" 0.82	5° 3" 50.59	" 1.40	35° 5" 38.03	" 2.	20° 1" 0.77	" 0.35	20° 1" 23.79	" 0.51	20° 2" 0.32	" 0.89	10° 3" 51.99	" 1.42	40° 5" 40.90	" 2.
30° 1" 1.13	" 0.36	30° 1" 24.31	" 0.52	30° 2" 1.21	" 0.89	40° 2" 53.65	" 0.84	15° 3" 53.41	" 1.43	45° 5" 43.81	" 2.	40° 1" 1.48	" 0.35	40° 1" 24.84	" 0.53	40° 2" 2.12	" 0.91	20° 3" 54.84	" 1.43	50° 5" 46.77	" 2.
50° 1" 1.84	" 0.36	50° 1" 25.37	" 0.54	50° 2" 3.03	" 0.93	55° 2" 56.18	" 0.85	25° 3" 56.29	" 1.46	55° 5" 49.78	" 3.	60° 0" 0" 0.07	" 0.35	55° 1" 22.77	" 0.54	64° 0" 1" 58.56	" 0.93	76° 0" 3" 49.21	" 1.46	30° 5" 35.21	" 3.
70° 0" 0" 1.20	" 0.37	56° 0" 1" 25.91	" 0.54	65° 0" 2" 3.96	" 0.94	72° 0" 2" 57.03	" 0.86	30° 3" 57.75	" 1.48	81° 0" 5" 52.83	" 3.	10° 1" 2.57	" 0.36	10° 1" 26.45	" 0.54	10° 2" 4.90	" 0.95	35° 3" 59.23	" 1.50	5° 5" 55.93	" 3.
20° 1" 2.93	" 0.37	20° 1" 26.99	" 0.55	20° 2" 5.85	" 0.96	10° 2" 58.76	" 0.88	40° 4" 0.73	" 1.52	10° 5" 59.09	" 3.	30° 1" 3.30	" 0.37	30° 1" 27.54	" 0.55	30° 2" 6.81	" 0.97	15° 2" 59.64	" 1.53	15° 6" 2.30	" 3.
40° 1" 3.67	" 0.37	40° 1" 28.09	" 0.56	40° 2" 7.78	" 0.99	20° 3" 0.53	" 0.90	50° 4" 3.78	" 1.55	20° 6" 5.56	" 3.	50° 1" 4.04	" 0.37	50° 1" 28.65	" 0.56	50° 2" 8.77	" 1.00	55° 4" 5.33	" 1.56	25° 6" 8.88	" 3.
80° 0" 0" 4.41	" 0.38	57° 0" 1" 29.21	" 0.57	66° 0" 2" 9.77	" 1.01	30° 3" 2.33	" 0.91	77° 0" 4" 6.89	" 1.59	30° 6" 12.26	" 3.	10° 1" 4.79	" 0.38	10° 1" 29.78	" 0.57	10° 2" 10.78	" 1.03	35° 4" 8.48	" 1.61	35° 6" 15.69	" 3.
20° 1" 5.17	" 0.38	20° 1" 30.35	" 0.58	20° 2" 11.81	" 1.04	40° 3" 4.16	" 0.92	10° 4" 10.09	" 1.63	40° 6" 19.19	" 3.	30° 1" 5.55	" 0.38	30° 1" 30.93	" 0.58	30° 2" 12.85	" 1.05	15° 4" 11.72	" 1.65	45° 6" 22.74	" 3.
40° 1" 5.94	" 0.39	40° 1" 31.51	" 0.59	40° 2" 13.90	" 1.07	50° 3" 6.02	" 0.93	20° 4" 13.37	" 1.66	50° 6" 26.36	" 3.	50° 1" 6.33	" 0.39	50° 1" 32.10	" 0.59	50° 2" 14.97	" 1.08	25° 4" 15.03	" 1.69	55° 6" 30.04	" 3.
90° 0" 0" 6.72	" 0.39	58° 0" 1" 32.69	" 0.60	67° 0" 2" 16.05	" 1.09	73° 0" 3" 7.92	" 0.96	30° 4" 16.72	" 1.71	82° 0" 6" 33.79	" 3.	10° 1" 7.11	" 0.39	10° 1" 33.29	" 0.61	10° 2" 17.14	" 1.11	35° 4" 18.43	" 1.73	5° 6" 37.61	" 3.
20° 1" 7.50	" 0.39	20° 1" 33.90	" 0.61	20° 2" 18.25	" 1.13	10° 3" 9.85	" 0.97	40° 4" 20.16	" 1.75	10° 6" 41.50	" 3.	30° 1" 7.90	" 0.40	30° 1" 34.51	" 0.62	30° 2" 19.38	" 1.14	15° 3" 10.83	" 1.91	15° 6" 45.46	" 4.
40° 1" 8.30	" 0.40	40° 1" 35.13	" 0.62	40° 2" 20.52	" 1.16	20° 3" 11.82	" 0.99	50° 4" 23.68	" 1.77	20° 6" 49.49	" 4.	50° 1" 8.70	" 0.40	50° 1" 35.75	" 0.63	50° 2" 21.68	" 1.17	25° 4" 25.48	" 1.82	25° 6" 53.59	" 4.
0° 0" 0" 9.11	" 0.41	59° 0" 1" 36.38	" 0.63	68° 0" 2" 22.85	" 1.19	30° 3" 13.82	" 1.02	78° 0" 4" 27.30	" 1.84	30° 6" 57.78	" 4.	10° 1" 9.52	" 0.41	10° 1" 37.01	" 0.64	10° 2" 24.04	" 1.21	35° 4" 29.14	" 1.87	35° 7" 2.04	" 4.
20° 1" 9.93	" 0.41	20° 1" 37.65	" 0.65	20° 2" 25.25	" 1.22	40° 3" 15.86	" 1.04	10° 4" 31.01	" 1.89	40° 7" 6.39	" 4.	20° 1" 10.34	" 0.42	30° 1" 38.30	" 0.65	30° 2" 26.47	" 1.24	45° 3" 32.90	" 1.91	45° 7" 10.82	" 4.
40° 1" 10.76	" 0.42	40° 1" 38.95	" 0.66	40° 2" 27.71	" 1.26	50° 3" 17.95	" 1.06	20° 4" 34.81	" 1.94	50° 7" 15.33	" 4.	50° 1" 11.18	" 0.42	50° 1" 39.51	" 0.67	50° 2" 28.97	" 1.28	25° 4" 36.75	" 1.97	55° 7" 19.93	" 4.
1° 0" 0" 11.60	" 0.43	60° 0" 1" 40.28	" 0.67	69° 0" 2" 30.25	" 1.30	74° 0" 3" 20.07	" 1.08	30° 4" 38.72	" 1.99	83° 0" 7" 24.63	" 4.	10° 1" 12.03	" 0.43	10° 1" 40.95	" 0.68	10° 2" 31.55	" 1.32	35° 4" 40.71	" 2.02	35° 7" 29.42	" 4.
20° 1" 12.46	" 0.43	20° 1" 41.63	" 0.69	20° 2" 32.87	" 1.34	10° 3" 22.23	" 1.08	40° 4" 42.73	" 2.05	40° 7" 34.30	" 4.	30° 1" 12.89	" 0.43	30° 1" 42.32	" 0.69	30° 2" 34.21	" 1.35	15° 3" 23.33	" 1.11	15° 7" 39.29	" 5.
40° 1" 13.33	" 0.44	40° 1" 43.01	" 0.70	40° 2" 35.56	" 1.38	20° 3" 24.44	" 1.12	50° 4" 46.85	" 2.11	20° 7" 44.38	" 5.	50° 1" 13.77	" 0.44	50° 1" 43.71	" 0.71	50° 2" 36.94	" 1.40	25° 4" 48.96	" 2.13	25° 7" 49.57	" 5.
2° 0" 0" 14.21	" 0.44	61° 0" 1" 44.42	" 0.72	70° 0" 2" 38.34	" 0.71	30° 3" 26.69	" 1.15	79° 0" 4" 51.09	" 2.16	30° 7" 54.87	" 5.	10° 1" 14.65	" 0.44	10° 1" 45.14	" 0.72	10° 2" 39.05	" 0.71	35° 4" 53.25	" 2.19	35° 8" 0.28	" 5.
20° 1" 15.10	" 0.45	20° 1" 45.86	" 0.73	10° 2" 39.76	" 0.72	40° 3" 28.99	" 1.17	10° 4" 55.44	" 2.23	40° 8" 5.80	" 5.	30° 1" 15.55	" 0.45	30° 1" 46.59	" 0.74	30° 2" 40.48	" 0.73	15° 4" 57.67	" 2.25	45° 8" 11.45	" 5.
40° 1" 16.01	" 0.46	40° 1" 47.33	" 0.75	20° 2" 41.21	" 0.73	50° 3" 31.34	" 1.18	20° 4" 59.92	" 2.29	50° 8" 17.21	" 5.	50° 1" 16.47	" 0.46	50° 1" 48.08	" 0.75	50° 2" 41.94	" 0.74	25° 5" 2.21	" 2.32	55° 8" 23.10	" 6.
3° 0" 0" 16.93	" 0.46	62° 0" 1" 48.83	" 0.76	30° 2" 42.68	" 0.74	75° 0" 3" 33.73	" 1.22	30° 5" 4.53	" 2.35	84° 0" 8" 29.13	" 6.	10° 1" 17.39	" 0.47	10° 1" 49.59	" 0.77	10° 2" 43.42	" 0.75	35° 5" 35.28	" 2.39	35° 8" 35.28	" 6.
20° 1" 17.86	" 0.47	20° 1" 50.36	" 0.78	40° 2" 44.17	" 0.76	10° 3" 36.18	" 1.23	40° 5" 6.88	" 2.42	40° 8" 41.57	" 6.	30° 1" 18.33	" 0.47	30° 1" 51.14	" 0.79	45° 2" 44.93	" 0.76	15° 3" 37.42	" 2.46	15° 8" 48.00	" 6.
40° 1" 18.81	" 0.48	40° 1" 51.93	" 0.80	50° 2" 45.69	" 0.77	20° 3" 38.67	" 1.24	45° 5" 11.69	" 2.46	20° 8" 54.57	" 6.	50° 1" 19.29	" 0.49	50° 1" 52.73	" 0.80	55° 2" 46.46	" 0.77	25° 5" 16.65	" 2.53	25° 9" 1.30	" 6.

duced, 00462 for each minute.

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lefrac.	Diff.	Zen. Dis.	Refrac.	Diff.	Zen. Dis.	Refrac.	Diff.
" 19.18	" 2.57	84° 30'	9° 8.18"	" 7.05	88° 18'	19° 54.47"	" 17.06
21.75	2.61	35°	9° 15.23	7.20	21° 20'	11.53	17.45
24.36	2.65	40°	9° 22.43	7.38	24° 20'	28.98	17.84
27.01	2.70	45°	9° 29.81	7.56	27° 20'	46.82	18.26
29.71	2.73	50°	9° 37.37	7.73	30° 21'	5.08	18.68
32.44	2.77	55°	9° 45.10	7.93	33° 21'	23.76	19.12
35.21	2.82	85°	9° 53.03	8.12	36° 21'	42.88	
38.03	2.87	5° 10'	1.15	8.31	39° 22'	2.45	19.57
40.90	2.91	10° 10'	9.46	8.53	42° 22'	22.47	20.02
43.81	2.96	15° 10'	17.99	8.74	45° 22'	42.96	20.49
46.77	3.01	20° 10'	26.73	8.96	48° 23'	3.95	20.99
49.78	3.05	25° 10'	35.69	9.19	51° 23'	25.42	21.47
52.83	3.10	30° 10'	44.88	9.43	54° 23'	47.40	22.50
55.93	3.16	35° 10'	54.31	9.67	57° 24'	9.90	23.04
59.09	3.21	40° 11'	3.98	9.92	89° 024'	32.94	15.66
2.30	3.26	45° 11'	13.90	10.19	2° 24'	48.60	15.91
5.56	3.32	50° 11'	24.09	10.45	4° 25'	4.51	16.16
8.88	3.38	55° 11'	34.54	10.73	6° 25'	20.67	16.42
12.26	3.43	86° 011'	45.27	8.80	8° 25'	37.09	16.68
15.69	3.49	4° 11'	54.07	8.99	10° 25'	53.77	16.94
19.19	3.50	8° 12'	3.06	9.18	12° 26'	10.71	17.20
22.74	3.55	12° 12'	12.24	9.39	14° 26'	27.91	17.49
26.36	3.62	16° 12'	21.63	9.59	16° 26'	45.40	17.76
30.04	3.68	20° 12'	31.22	9.80	18° 27'	3.16	18.04
33.79	3.82	24° 12'	41.02	10.02	20° 27'	21.20	18.32
37.61	3.89	28° 12'	51.04	10.24	22° 27'	39.52	18.62
41.50	3.96	32° 13'	1.28	10.47	24° 27'	58.14	18.91
45.46	4.03	36° 13'	11.75	10.71	26° 28'	17.05	19.21
49.49	4.10	40° 13'	22.46	10.96	28° 28'	36.26	
53.59	4.19	44° 13'	33.42	11.20	30° 28'	55.78	19.52
57.78	4.26	48° 13'	44.62	11.46	32° 29'	15.60	20.14
2.04	4.35	52° 13'	56.08	11.73	34° 29'	35.74	20.45
6.39	4.35	56° 14'	7.81	11.99	36° 29'	56.19	20.78
10.82	4.43	87° 014'	19.80	13.11	38° 30'	16.97	21.10
15.33	4.51	4° 14'	32.91	13.46	40° 30'	38.07	21.44
19.93	4.60	8° 14'	46.37	13.83	42° 30'	59.51	21.77
24.63	4.79	12° 15'	0.20	14.20	44° 31'	21.28	22.11
29.42	4.88	16° 15'	14.40	14.60	46° 31'	43.39	22.46
34.30	4.99	20° 15'	29.00	15.00	48° 32'	5.85	22.81
39.29	5.09	24° 15'	44.00	15.42	50° 32'	28.66	23.16
44.38	5.19	28° 15'	59.42	15.85	52° 32'	51.82	23.53
49.57	5.30	32° 16'	15.27	16.31	54° 33'	15.35	23.89
54.87	5.41	36° 16'	31.58	16.77	56° 33'	39.24	24.26
0.28	5.41	40° 16'	48.35	17.26	58° 34'	3.5	24.63
5.80	5.52	44° 17'	5.61	17.76	90° 034'	28.13	25.02
11.45	5.65	48° 17'	23.37	18.27	2° 34'	53.15	25.40
17.21	5.76	52° 17'	41.64	18.82	4° 35'	18.55	25.79
23.10	5.89	56° 18'	0.46	19.37	6° 35'	44.34	26.18
29.13	6.15	88° 018'	19.83	14.91	8° 36'	10.52	26.58
35.28	6.29	3° 18'	34.74	15.24	10° 36'	37.10	26.98
41.57	6.43	6° 18'	49.98	15.59	12° 37'	4.08	27.39
48.00	6.43	9° 19'	5.57	15.94	14° 37'	31.47	
54.57	6.57	12° 19'	21.51	16.29	16° 37'	59.28	27.81
1.30	6.73	15° 19'	37.80	16.67	18° 38'	27.50	28.22

Barometer
in inches.

FAHRENHEIT's Thermometer.

	Barometer			FAHRENHEIT's Thermometer.		
	Within.		Without.	Within.		Without.
	°	+	°	—	—	—
28.60	,0350	29.60	,0000	21.5	,0632	,0470
62	,0342	62	,0007	22.0	,0621	,0460
64	,0335	64	,0014	22.5	,0609	,0450
66	,0328	66	,0020	23.0	,0598	,0440
68	,0321	68	,0027	23.5	,0586	,0430
28.70	,0314	29.70	,0034	24.0	,0575	,0420
72	,0306	72	,0041	24.5	,0563	,0410
74	,0299	74	,0047	25.0	,0552	,0400
76	,0292	76	,0054	25.5	,0540	,0390
78	,0285	78	,0061	26.0	,0529	,0380
28.80	,0278	29.80	,0068	26.5	,0517	,0370
82	,0271	82	,0074	27.0	,0506	,0360
84	,0264	84	,0081	27.5	,0494	,0350
86	,0256	86	,0088	28.0	,0483	,0340
88	,0249	88	,0095	28.5	,0471	,0330
28.90	,0242	29.90	,0101	29.0	,0460	,0320
92	,0235	92	,0108	29.5	,0448	,0310
94	,0228	94	,0115	30.0	,0437	,0300
96	,0221	96	,0122	30.5	,0425	,0290
98	,0214	98	,0128	31.0	,0414	,0280
29.00	,0207	30.00	,0135	31.5	,0402	,0270
02	,0200	02	,0142	32.0	,0391	,0260
04	,0193	04	,0149	32.5	,0379	,0250
06	,0186	06	,0155	33.0	,0368	,0240
08	,0179	08	,0162	33.5	,0356	,0230
29.10	,0172	30.10	,0169	34.0	,0345	,0220
12	,0165	12	,0176	34.5	,0333	,0210
14	,0158	14	,0182	35.0	,0322	,0200
16	,0151	16	,0189	35.5	,0310	,0190
18	,0144	18	,0196	36.0	,0299	,0180
29.20	,0137	30.20	,0203	36.5	,0287	,0170
22	,0130	22	,0210	37.0	,0276	,0160
24	,0123	24	,0216	37.5	,0264	,0150
26	,0116	26	,0223	38.0	,0253	,0140
28	,0109	28	,0230	38.5	,0241	,0130
29.30	,0102	30.30	,0237	39.0	,0230	,0120
32	,0096	32	,0243	39.5	,0218	,0110
34	,0089	34	,0250	40.0	,0207	,0100
36	,0082	36	,0257	40.5	,0195	,0090
38	,0075	38	,0264	41.0	,0184	,0080
29.40	,0068	30.40	,0270	41.5	,0172	,0070
42	,0061	42	,0277	42.0	,0161	,0060
44	,0054	44	,0284	42.5	,0149	,0050
46	,0048	46	,0291	43.0	,0138	,0040
48	,0041	48	,0297	43.5	,0126	,0030
29.50	,0034	30.50	,0304	44.0	,0115	,0020
52	,0027	52	,0311	44.5	,0103	,0010
54	,0020	54	,0318	45.0	,0092	—
56	,0014	56	,0324	45.5	,0080	,0010
58	,0007	58	,0331	46.0	,0069	,0020
				46.5	,0057	,0030
				47.0	,0046	,0040
				47.5	,0034	,0050
				48.0	,0023	,0060
				48.5	,0011	,0070

Factors for the correction of the mean Refraction.

Mean Refraction calculated at $58^{\circ} 13' 29.67$ for 45 Degrees of apparent Zenith Distance \times Tang. Z — $3.6342956r$ so far as 87° Zen. Dis. below which r is reduced, 00462 for each minute.

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Zen. Dis.	Refrac.	Diff.	Zen. Dis.	Refrac.	Diff.	Zen. Dis.	Refrac.	Diff.	Zen. Dis.	Refrac.	Diff.	Zen. Dis.	Refrac.	Diff.	Zen. Dis.	Refrac.	Diff.	Zen. Dis.	Refrac.	Diff.	Zen. Dis.	Refrac.	Diff.	Zen. Dis.	Refrac.	Diff.
0° 0' 0" 0.00	0.17	9° 0' 0" 9.20	0.17	18° 0' 0" 18.86	0.19	27° 0' 0" 29.58	0.22	36° 0' 0" 42.17	0.26	45° 0' 0" 58.01	0.34	54° 0' 0" 19.78	0.49	63° 0' 0" 53.53	0.82	71° 0' 0" 24.73	0.78	75° 30' 0" 41.22	1.30	80° 0' 0" 19.18	0.84	84° 30' 0" 8.18	7.05	88° 18' 0" 54.47	17.05	
10° 0' 0" 0.17	10° 0' 0" 9.37	0.17	10° 0' 0" 19.05	0.19	10° 0' 0" 29.80	0.20	10° 0' 0" 42.43	0.26	10° 0' 0" 58.35	0.34	10° 0' 0" 20.27	0.49	10° 0' 0" 54.35	0.82	10° 0' 0" 24.80	0.79	35° 3 42.52	1.31	5 21.75	3.5	9 15.23	7.20	21 20 11.53	17.45		
20° 0' 0" 0.34	20° 0' 0" 9.54	0.18	20° 0' 0" 19.24	0.19	20° 0' 0" 30.00	0.22	20° 0' 0" 42.69	0.26	20° 0' 0" 58.69	0.34	20° 0' 0" 20.76	0.49	20° 0' 0" 55.17	0.83	20° 0' 0" 24.80	0.79	40° 3 43.83	1.32	10° 5 24.36	2.65	9 22.43	7.38	24 20 28.98	17.84		
30° 0' 0" 0.51	30° 0' 0" 9.72	0.17	30° 0' 0" 19.43	0.18	30° 0' 0" 30.22	0.22	30° 0' 0" 42.95	0.26	30° 0' 0" 59.03	0.34	30° 0' 0" 21.25	0.49	30° 0' 0" 56.00	0.85	15° 2 49.59	0.80	45° 3 45.15	1.34	15° 5 27.01	4.5	9 29.81	7.56	27 20 46.82	18.26		
40° 0' 0" 0.68	40° 0' 0" 9.89	0.17	40° 0' 0" 19.61	0.18	40° 0' 0" 30.44	0.22	40° 0' 0" 43.21	0.26	40° 0' 0" 59.38	0.35	40° 0' 0" 21.75	0.50	40° 0' 0" 56.85	0.85	20° 2 50.39	0.81	50° 3 46.49	1.35	20° 5 29.71	2.73	50° 9 37.37	7.73	30 21 5.08	18.68		
50° 0' 0" 0.84	50° 0' 0" 10.06	0.18	50° 0' 0" 19.80	0.19	50° 0' 0" 30.65	0.22	50° 0' 0" 43.47	0.27	50° 0' 0" 59.72	0.35	50° 0' 0" 22.26	0.51	50° 0' 0" 57.70	0.86	25° 2 51.20	0.81	55° 3 47.84	1.37	25° 5 32.44	2.77	55° 9 45.10	7.93	33 21 23.76	19.12		
1° 0' 0" 1.01	10° 0' 0" 10.24	0.17	19° 0' 0" 19.99	0.19	28° 0' 0" 30.87	0.22	37° 0' 0" 43.74	0.26	46° 0' 0" 1.07	0.35	55° 0' 0" 22.77	0.51	64° 0' 0" 58.56	0.87	30° 2 52.01	0.82	76° 0' 0" 3 49.21	1.38	30° 5 35.21	2.82	85° 0' 0" 9.53	8.12	36 21 42.88	19.57		
10° 0' 0" 1.18	10° 0' 0" 10.41	0.18	10° 0' 0" 20.18	0.19	10° 0' 0" 31.09	0.21	10° 0' 0" 44.00	0.27	10° 0' 0" 59.43	0.51	10° 0' 0" 23.28	0.89	35° 2 52.83	0.82	5 3 50.59	1.40	35° 5 38.03	2.87	5 10 1.15	8.31	39 22 2.45	20.02				
20° 0' 0" 1.35	20° 0' 0" 10.59	0.17	20° 0' 0" 20.37	0.19	20° 0' 0" 31.30	0.22	20° 0' 0" 44.27	0.26	20° 0' 0" 57.77	0.56	20° 0' 0" 23.79	0.89	40° 2 53.65	0.84	10° 3 51.99	1.42	40° 5 40.90	2.91	10 10 9.46	8.53	42 22 22.47	20.49				
30° 0' 0" 1.52	30° 0' 0" 10.76	0.18	30° 0' 0" 20.56	0.19	30° 0' 0" 31.52	0.22	30° 0' 0" 44.53	0.27	30° 0' 0" 6.13	0.53	30° 0' 0" 24.31	0.91	45° 2 54.49	0.84	15° 3 53.41	1.43	45° 5 43.81	2.96	15 10 17.99	8.74	45 22 42.96	20.99				
40° 0' 0" 1.69	40° 0' 0" 10.94	0.17	40° 0' 0" 20.75	0.19	40° 0' 0" 31.74	0.22	40° 0' 0" 44.80	0.27	40° 0' 0" 1.48	0.56	40° 0' 0" 24.84	0.91	50° 2 55.33	0.85	20° 3 54.84	1.45	50° 5 46.77	3.01	20 10 26.73	8.96	48 23 3.95	21.47				
50° 0' 0" 1.86	50° 0' 0" 11.11	0.18	50° 0' 0" 20.94	0.19	50° 0' 0" 31.96	0.22	50° 0' 0" 45.07	0.27	50° 0' 0" 1.84	0.36	50° 0' 0" 25.37	0.54	55° 2 50.18	0.85	25° 3 56.29	1.46	55° 5 49.78	3.05	25 10 35.69	9.19	51 23 25.42	21.98				
2° 0' 0" 2.03	11° 0' 0" 11.29	0.17	20° 0' 0" 21.13	0.19	29° 0' 0" 32.18	0.22	38° 0' 0" 45.34	0.28	47° 0' 0" 2.20	0.37	56° 0' 0" 25.91	0.54	65° 0' 0" 3.96	0.94	72° 0' 0" 57.03	0.86	30° 3 57.75	1.48	81° 0' 0" 5 52.83	3.10	30 10 44.88	9.43	54 23 47.40	22.50		
10° 0' 0" 2.20	10° 0' 0" 11.46	0.18	10° 0' 0" 21.32	0.19	10° 0' 0" 32.40	0.22	10° 0' 0" 45.62	0.27	10° 0' 0" 2.57	0.36	10° 0' 0" 26.45	0.54	10° 0' 0" 4.90	0.95	5 2 57.89	0.87	35° 3 59.23	1.50	35 10 54.31	9.67	57 24 9.90	23.04				
20° 0' 0" 2.37	20° 0' 0" 11.64	0.17	20° 0' 0" 21.51	0.20	20° 0' 0" 32.62	0.23	20° 0' 0" 45.89	0.27	20° 0' 0" 2.93	0.37	20° 0' 0" 26.99	0.55	20° 0' 0" 5.85	0.96	40° 4 0.73	1.52	10 5 59.09	2.21	40 11 3.98	9.92	89 0 24 32.94	15.66				
30° 0' 0" 2.53	30° 0' 0" 11.81	0.17	30° 0' 0" 21.71	0.19	30° 0' 0" 32.85	0.23	30° 0' 0" 46.16	0.28	30° 0' 0" 3.30	0.37	30° 0' 0" 27.54	0.55	30° 0' 0" 6.81	0.97	15° 2 59.64	0.89	45° 4 2.25	1.52	15 6 2.30	2.36	45 11 13.90	10.19	24 24 48.60	15.66		
40° 0' 0" 2.70	40° 0' 0" 11.99	0.18	40° 0' 0" 21.90	0.19	40° 0' 0" 33.07	0.22	40° 0' 0" 46.44	0.28	40° 0' 0" 3.67	0.37	40° 0' 0" 28.09	0.56	40° 0' 0" 7.8	0.99	50° 3 0.53	0.90	50° 4 3.78	1.55	20 6 5.56	3.22	50 11 24.09	10.45	45 22 45.51	16.16		
50° 0' 0" 2.87	50° 0' 0" 12.17	0.17	50° 0' 0" 22.09	0.20	50° 0' 0" 33.29	0.23	50° 0' 0" 46.72	0.28	50° 0' 0" 4.04	0.37	50° 0' 0" 28.65	0.56	50° 0' 0" 8.77	1.00	25° 3 1.43	0.90	55° 4 5.33	1.56	25 6 8.88	3.38	55 11 34.54	10.73	62 25 20.67	16.42		
3° 0' 0" 3.04	12° 0' 0" 12.34	0.18	21° 0' 0" 22.29	0.19	30° 0' 0" 33.52	0.22	39° 0' 0" 47.00	0.28	48° 0' 0" 4.41	0.38	57° 0' 0" 29.31	0.57	66° 0' 0" 9.77	1.01	77° 0' 0" 3 2.33	0.91	77° 0' 0" 4 6.89	1.59	30° 6 12.26	3.43	86° 0' 0" 11 45.27	8.80	82 25 37.09	16.68		
10° 0' 0" 3.21	10° 0' 0" 12.52	0.18	10° 0' 0" 22.48	0.19	10° 0' 0" 33.74	0.23	10° 0' 0" 47.28	0.28	10° 0' 0" 4.79	0.38	10° 0' 0" 29.78	0.57	10° 0' 0" 10.78	1.03	35° 3 3.24	0.92	35 4 8.48	1.59	35 6 15.69	3.50	41 15 54.07	8.99	10 25 53.77	16.94		
20° 0' 0" 3.38	20° 0' 0" 12.70	0.17	20° 0' 0" 22.67	0.20	20° 0' 0" 33.97	0.23	20° 0' 0" 47.56	0.28																		